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DOUBLE-PERIODIC AND MULTIPLE SOLITON SOLUTIONS FOR SOME NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

ABSTRACT

In this paper we implemented generalized Jacobi elliptic function method with symbolic computation to construct new double-periodic and multiple soliton solutions for KdV equation and (2+1)-dimensional coupled KdV system.

Keywords: KdV Equation, (2+1)-Dimensional Coupled Kdv System, Generalized Jacobi Elliptic Function Method, Double-Periodic Solutions, Multiple Soliton Solutions

BAZI LİNEER OLMAYAN KISMİ DİFERENSİYEL DENKLEMLER İÇİN YENİ PERİYODİK VE ÇOK KATLI SOLİTON ÇÖZÜMLER

ÖZET

Bu çalışmada biz sembolik bilgisayar programı yardımı ile genelleştirilmiş jaccobi eliptik fonksiyon metodunu kullanarak KdV denklemi ve (2+1)- boyutlu çift KdV sistemi için çok katlı soliton çözümler ve yeni periyodik çözümler sunarız.

Anahtar Kelimeler: KdV Denklemi, (2+1)- Boyutlu Çift KdV Sistemi, Genelleştirilmiş Jaccobi Eliptik Fonksiyon Metot, Çift Periyodik Çözümler, Çok Katlı Soliton Çözümler

1. INTRODUCTION (GİRİŞ)

The theory of nonlinear dispersive wave motion has recently undergone much study. We do not attempt to characterize the general form of nonlinear dispersive wave equations [1 and 2]. Nonlinear phenomena play a crucial role in applied mathematics and physics. Furthermore, when an original nonlinear equation is directly calculated, the solution will preserve the actual physical characters of solutions [3]. Explicit solutions to the nonlinear equations are of fundamental importance. Various methods for obtaining explicit solutions to nonlinear evolution equations have been proposed. Many explicit exact methods have been introduced in literature [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23 ve 24]. Among them are Generalized Miura Transformation, Darboux Transformation, Cole-Hopf Transformation, Hirota's dependent variable Transformation, the inverse scattering Transform and the Backlund Transformation, tanh method, sine-cosine method, Painleve method, homogeneous balance method, similarity reduction method, improved tanh method and so on. In fact, recently a direct algebraic approach has been constructed an automated tanh-function method by Parkes and Duffy [12]. The authors present a Mathematica package that deals with complicated algebraic and outputs directly the required solutions for particular nonlinear equations.

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this study, we implemented a generalized Jacobi elliptic function method [24] with symbolic computation to construct new double-periodic solutions and multiple soliton solutions for KdV equation and (2+1)-dimensional coupled KdV system.

3. METHOD AND ITS APPLICATIONS (YÖNTEM VE UYGULAMALARI)

Before starting to give a generalized Jacobi elliptic function method [24], we will give a simple description of the generalized Jacobi-function method. For doing this, one can consider in a two variables general form of nonlinear PDE

$$Q(u, u_t, u_x, u_{xx}, \ldots) = 0\ , \tag{1}$$

and transform Eq.(1) with $u(x,t) = u(\xi),\ \xi = x + wt$, and $\xi = x + \beta y + \alpha t,$ where w, α, β is a constant. After transformation, we get a nonlinear ODE for $u(\xi)$

$$Q'(u', u'', u''', \ldots) = 0. \tag{2}$$

The solution of the equation (2) we are looking for is expressed in the form

$$U_i(\xi) = a_0 + \sum_{i=1}^{n}\left[a_i F^i(\xi) + b_i F^{-i}(\xi)\right], \tag{3}$$

where $\xi = x + wt$ and $\xi = x + \beta y + \alpha t,$ n is a positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation, $\alpha, \beta, a_0, a_i, b_i$ and ξ can be determined. Substituting solution (3) into Eq. (2) yields a set of algebraic equations for $F^i\left(\sqrt{A + BF^2 + CF^4}\right)^j$, $(j = 0,1,\ldots)$ and $(i = 0,1,2,\ldots)$ then, all coefficients of $F^i\left(\sqrt{A + BF^2 + CF^4}\right)^j$ have to vanish. After this separated algebraic equation, we could found coefficients $\alpha, \beta, a_0, a_i, b_i$ and ξ .

In this work, we will consider to solve the KdV equation and (2+1)-dimensional coupled KdV system by using the generalized Jacobi elliptic function method which is introduced by Huai-Tang and Hong-Qing [24]. The fundamental of their method is to take full advantage of the elliptic equation and use its solutions *F*. The desired elliptic equation is given as

$$F'^2 = A + BF^2 + CF^4 , \qquad (4)$$

where $F' = \frac{dF}{d\xi}$ and a,b,c are constants. Some of the solutions are given the paper [24]. In this study we have given several extra cases so that we have obtained double-periodic solutions and multiple soliton solutions of Eq. (2) in the form of Jacobi elliptic functions (4).

4. EXAMPLE 1. (ÖRNEK 1)

Consider a KdV equation,

$$u_t + u_x + uu_x + u_{xxx} = 0, \qquad (5)$$

For doing this example, we can use transformation with Eq(1). then Eq(5), become

$$wu' + u' + uu' + u''' = 0, \qquad (6)$$

and integrating (6) yields, we yield following equation

$$wu + u + \frac{u^2}{2} + u'' = 0, \qquad (7)$$

When balancing u^2 with u'' then gives *n=2*. Therefore, we may choose

$$u = a_0 + a_1 F + a_2 F^2 + \frac{b_1}{F} + \frac{b_2}{F^2} \ . \qquad (8)$$

Substituting (8) into Eq. (7) yields a set of algebraic equations for w, a_0, a_i, b_i . These systems are finding as

$$a_0 + \frac{a_0^2}{2} + 2Aa_2 + a_1 b_1 + a_2 b_2 + 2b_2 C + a_0 w = 0,$$

$$6Ab_2 + \frac{b_2^2}{2} = 0, \quad 2Ab_1 + b_1 b_2 + \frac{b_1^2}{2} + b_2 + a_0 b_2 + 4Bb_2 + b_2 w = 0,$$

$$b_1 + a_0 b_1 + Bb_1 + a_1 b_2 + b_1 w = 0, \ a_1 + a_0 a_1 + a_1 B + a_2 b_1 + a_1 w = 0,$$

$$\frac{a_1^2}{2} + a_2 + a_0 a_2 + 4a_2 B + a_2 w = 0, \ a_1 a_2 + 2a_1 C = 0, \ \frac{a_2^2}{2} + 6a_2 C = 0. \qquad (9)$$

From the solutions of the system, we can found

$$a_0 = -4\left(B + \sqrt{B^2 - 3AC}\right), \ a_1 = 0, \ a_2 = -12C, \ b_1 = 0, \ b_2 = 0, \ w = -1 + 4\sqrt{B^2 - 3AC}. \qquad (10)$$

with the aid of Mathematica.

Substituting (10) into (8) we have obtained the following double-periodic solutions of equation (5). These solutions are:

i) If $A = 1, \ B = -\left(1 + m^2\right), \ C = m^2$. (11)

$$u_1 = -4\left(B + \sqrt{B^2 - 3AC}\right) - 12Csn^2(\xi) \ .$$

$$u_2 = -4\left(B + \sqrt{B^2 - 3AC}\right) - 12C\left(\frac{cn(\xi)}{dn(\xi)}\right)^2 \ .$$

ii) If $A = 1 - m^2 , \ B = 2m^2 - 1, \ C = -m^2$. (12)

$$u_3 = -4\left(B + \sqrt{B^2 - 3AC}\right) - 12Ccn^2(\xi) \quad .$$

$iii)$ If $A=m^2-1,\ B=2-m^2,\ C=-1$. (13)

$$u_4=-4\left(B+\sqrt{B^2-3AC}\right)-12Cdn^2(\xi).$$

$iv)$ If $A=-m^2(1-m^2),\ B=2m^2-1,\ C=1$. (14)

$$u_5=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{dn(\xi)}{sn(\xi)}\right)^2.$$

$v)$ If $A=1-m^2,\ B=2-m^2,\ C=1$. (15)

$$u_6=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{cn(\xi)}{sn(\xi)}\right)^2.$$

$vi)$ If $A=\frac{1}{4},\ B=\frac{m^2-2}{2},\ C=\frac{m^2}{4}$. (16)

$$u_7=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{sn(\xi)}{1\pm dn(\xi)}\right)^2.$$

$vii)$ If $A=\frac{m^2}{4},\ B=\frac{m^2-2}{2},\ C=\frac{m^2}{4}$. (17)

$$u_8=-4\left(B+\sqrt{B^2-3AC}\right)-12C(sn(\xi)\pm icn(\xi))^2.$$

$$u_9=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{dn(\xi)}{i\sqrt{1-m^2}\,sn(\xi)\pm cn(\xi)}\right)^2.$$

$viii)$ If $A=\frac{1}{4},\ B=\frac{1-2m^2}{2},\ C=\frac{1}{4}$. (18)

$$u_{10}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{dn(\xi)}{mcn(\xi)\pm i\sqrt{1-m^2}}\right)^2.$$

$$u_{11}=-4\left(B+\sqrt{B^2-3AC}\right)-12C(msn(\xi)\pm idn(\xi))^2.$$

$$u_{12}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{sn(\xi)}{1\pm cn(\xi)}\right)^2.$$

$$u_{13}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{cn(\xi)}{\sqrt{1-m^2}\,sn(\xi)\pm dn(\xi)}\right)^2.$$

$ix)$ If $A=\frac{m^2-1}{4},\ B=\frac{m^2+1}{2},\ C=\frac{m^2-1}{4}$. (19)

$$u_{14}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{dn(\xi)}{1\pm msn(\xi)}\right)^2.$$

$x)$ If $A=\frac{1-m^2}{4},\ B=\frac{m^2+1}{2},\ C=\frac{1-m^2}{4}$. (20)

$$u_{15}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{cn(\xi)}{1\pm sn(\xi)}\right)^2.$$

$xi)$ If $A=-\frac{\left(1-m^2\right)^2}{4},\ B=\frac{m^2+1}{2},\ C=-\frac{1}{4}$. (21)

$$u_{16}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(mcn(\xi)\pm dn(\xi)\right)^2.$$

$xii)$ If $A=\frac{1}{4},\ B=\frac{m^2+1}{2},\ C=\frac{\left(1-m^2\right)^2}{4}$. (22)

$$u_{17}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{sn(\xi)}{dn(\xi)\pm cn(\xi)}\right)^2.$$

$xiii)$ If $A=\frac{1}{4},\ B=\frac{m^2-2}{2},\ C=\frac{m^4}{4}$. (23)

$$u_{18}=-4\left(B+\sqrt{B^2-3AC}\right)-12C\left(\frac{cn(\xi)}{\sqrt{1-m^2}\pm dn(\xi)}\right)^2.$$

where, $\xi=x+\left(-1+4\sqrt{B^2-3AC}\right)t$.

Here $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ are Jacobi elliptic functions and m denotes the modulus of the Jacobi elliptic functions.
If $m\to 1,$ then $sn\xi\to\tanh\xi,\quad cn\xi\to\sec h\xi,\quad dn\xi\to\sec h\xi.$ We can obtain the following some multiple soliton solutions of Eq.(5).

$$u_{19}=4-12\tanh^2(\xi)$$

$$u_{20}=-8+12\sec h^2(\xi)$$

where, $\xi=x+3t$.

If $m\to 0,$ then $sn\xi\to\sin\xi,\quad cn\xi\to\cos\xi,\quad dn\xi\to 1.$ We can obtain triangular periodic solutions of Eq.(5). The triangular periodic solutions of Eq. (5) are omitted.

5. EXAMPLE 2. (ÖRNEK 2)

Consider a (2+1)-dimensional coupled KdV equation,

$$u_t+u_{xxx}-3u_xv-3v_xu=0,\qquad u_x-v_y=0, \qquad (24)$$

For doing this example, we can use transformation with Eq(1), then Eq(24), become

$$\alpha u'+u'''-3u'v-3v'u=0,\qquad u'-\beta v=0, \qquad (25)$$

and integrating (25) yields, we yield following equation

$$\alpha u+u''-3uv=0,\qquad u-\beta v=0, \qquad (26)$$

When balancing uv with u'' and u with v then gives $n_1=2$ and $n_2=2$. Therefore, we may choose

$$u=a_0+a_1F+a_2F^2+\frac{b_1}{F}+\frac{b_2}{F^2}.$$

$$v=c_0+c_1F+c_2F^2+\frac{d_1}{F}+\frac{d_2}{F^2} \qquad (27)$$

Substituting (27) into Eq. (26) yields a set of algebraic equations for $\alpha, \beta, a_0, c_0, a_i, b_i, c_i, d_i$. These systems are finding as

$$a_0\alpha + 2Aa_2 + 2b_2C - 3a_0c_0 - 3c_1b_1 - 3c_2b_2 - 3a_1d_1 - 3a_2d_2 = 0,$$
$$a_1B + a_1\alpha - 3a_1c_0 - 3a_0c_1 - 3c_2b_1 - 3a_2d_1 = 0,$$
$$a_2\alpha + 4a_2B - 3a_2c_0 - 3a_1c_1 - 3a_0c_2 = 0, -3a_2c_1 - 3a_1c_2 + 2a_1C = 0,$$
$$-3a_2c_2 + 6a_2C = 0, b_1\alpha + Bb_1 - 3c_0b_1 - 3c_1b_2 - 3a_0d_1 - 3a_1d_2 = 0,$$
$$b_2\alpha - 3c_0b_2 - 3b_1d_1 - 3a_0d_2 + 4Bb_2 = 0, -3b_2d_1 - 3b_1d_2 + 2Ab_1 = 0, 6Ab_2 - 3b_2d_2 = 0,$$
$$a_0 - c_0\beta = 0, -c_1\beta + a_1 = 0, a_2 - c_2\beta = 0, b_1 - d_1\beta = 0, b_2 - d_2\beta = 0. \quad (28)$$

From the solutions of the system, we can found

$$a_0 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A}, \quad a_1 = 0, \quad a_2 = \frac{b_2C}{A}, \quad b_1 = 0, \quad b_2 = b_2, \quad c_0 = \frac{2}{3}(B + \sqrt{B^2 + 12AC}) \quad (29)$$

$$c_1 = 0, \quad c_2 = 2C, \quad d_1 = 0, \quad d_2 = 2A, \quad \alpha = 4\sqrt{B^2 + 12AC}, \quad \beta = \frac{b_2}{2A}, \quad b_2 \neq 0.$$

with the aid of Mathematica.

Substituting (29) into (27) we have obtained the following double-periodic solutions of equation (24). These solutions are:

$i)$ If $A = 1,\ B = -(1 + m^2),\ C = m^2.$ (30)

$$u_1 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}sn^2(\xi) + b_2\left(\frac{1}{sn^2(\xi)}\right).$$

$$v_1 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2Csn^2(\xi) + 2A\left(\frac{1}{sn^2(\xi)}\right).$$

$$u_2 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{cn(\xi)}{dn(\xi)}\right)^2 + b_2\left(\frac{dn(\xi)}{cn(\xi)}\right)^2.$$

$$v_2 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{cn(\xi)}{dn(\xi)}\right)^2 + 2A\left(\frac{dn(\xi)}{cn(\xi)}\right)^2.$$

$ii)$ If $A = 1 - m^2,\ B = 2m^2 - 1,\ C = -m^2.$ (31)

$$u_3 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}cn^2(\xi) + b_2\left(\frac{1}{cn^2(\xi)}\right).$$

$$v_3 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2Ccn^2(\xi) + 2A\left(\frac{1}{cn^2(\xi)}\right).$$

$iii)$ If $A = m^2 - 1,\ B = 2 - m^2,\ C = -1.$ (32)

$$u_4 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}dn^2(\xi) + b_2\left(\frac{1}{dn^2(\xi)}\right).$$

$$v_4 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2Cdn^2(\xi) + 2A\left(\frac{1}{dn^2(\xi)}\right).$$

$iv)$ If $A = -m^2(1 - m^2),\ B = 2m^2 - 1,\ C = 1.$ (33)

$$u_5 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{dn(\xi)}{sn(\xi)}\right)^2 + b_2\left(\frac{sn(\xi)}{dn(\xi)}\right)^2.$$

$$v_5 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{dn(\xi)}{sn(\xi)}\right)^2 + 2A\left(\frac{sn(\xi)}{dn(\xi)}\right)^2.$$

$v)$ If $A = 1 - m^2$, $B = 2 - m^2$, $C = 1.$. (34)

$$u_6 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{cn(\xi)}{sn(\xi)}\right)^2 + b_2\left(\frac{sn(\xi)}{cn(\xi)}\right)^2.$$

$$v_6 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{cn(\xi)}{sn(\xi)}\right)^2 + 2A\left(\frac{sn(\xi)}{cn(\xi)}\right)^2.$$

$vi)$ If $A = \frac{1}{4}$, $B = \frac{m^2 - 2}{2}$, $C = \frac{m^2}{4}$. (35)

$$u_7 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{sn(\xi)}{1 \pm dn(\xi)}\right)^2 + b_2\left(\frac{1 \pm dn(\xi)}{sn(\xi)}\right)^2.$$

$$v_7 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{sn(\xi)}{1 \pm dn(\xi)}\right)^2 + 2A\left(\frac{1 \pm dn(\xi)}{sn(\xi)}\right)^2.$$

$vii)$ If $A = \frac{m^2}{4}$, $B = \frac{m^2 - 2}{2}$, $C = \frac{m^2}{4}$. (36)

$$u_8 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}(sn(\xi) \pm icn(\xi))^2 + b_2\left(\frac{1}{sn(\xi) \pm icn(\xi)}\right)^2.$$

$$v_8 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C(sn(\xi) \pm icn(\xi))^2 + 2A\left(\frac{1}{sn(\xi) \pm icn(\xi)}\right)^2.$$

$$u_9 = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{dn(\xi)}{i\sqrt{1 - m^2}\,sn(\xi) \pm cn(\xi)}\right)^2 + b_2\left(\frac{i\sqrt{1 - m^2}\,sn(\xi) \pm cn(\xi)}{dn(\xi)}\right)^2.$$

$$v_9 = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{dn(\xi)}{i\sqrt{1 - m^2}\,sn(\xi) \pm cn(\xi)}\right)^2 + 2A\left(\frac{i\sqrt{1 - m^2}\,sn(\xi) \pm cn(\xi)}{dn(\xi)}\right)^2.$$

$viii)$ If $A = \frac{1}{4}$, $B = \frac{1 - 2m^2}{2}$, $C = \frac{1}{4}$, $\xi = x + \alpha y - \left(4b\sqrt{B^2 - 3AC}\right)t$. (37)

$$u_{10} = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}\left(\frac{dn(\xi)}{mcn(\xi) \pm i\sqrt{1 - m^2}}\right)^2 + b_2\left(\frac{mcn(\xi) \pm i\sqrt{1 - m^2}}{dn(\xi)}\right)^2.$$

$$v_{10} = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{dn(\xi)}{mcn(\xi) \pm i\sqrt{1 - m^2}}\right)^2 + 2A\left(\frac{mcn(\xi) \pm i\sqrt{1 - m^2}}{dn(\xi)}\right)^2.$$

$$u_{11} = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2C}{A}(msn(\xi) \pm idn(\xi))^2 + b_2\left(\frac{1}{msn(\xi) \pm idn(\xi)}\right)^2.$$

$$v_{11} = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C(msn(\xi) \pm idn(\xi))^2 + 2A\left(\frac{1}{msn(\xi) \pm idn(\xi)}\right)^2.$$

$$u_{12}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}\left(\frac{sn(\xi)}{1\pm cn(\xi)}\right)^2+b_2\left(\frac{1\pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$v_{12}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C\left(\frac{sn(\xi)}{1\pm cn(\xi)}\right)^2+2A\left(\frac{1\pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$u_{13}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}\left(\frac{cn(\xi)}{\sqrt{1-m^2}sn(\xi)\pm dn(\xi)}\right)^2+b_2\left(\frac{\sqrt{1-m^2}sn(\xi)\pm dn(\xi)}{cn(\xi)}\right)^2.$$

$$v_{13}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C\left(\frac{cn(\xi)}{\sqrt{1-m^2}sn(\xi)\pm dn(\xi)}\right)^2+2A\left(\frac{\sqrt{1-m^2}sn(\xi)\pm dn(\xi)}{cn(\xi)}\right)^2.$$

ix) If $A=\frac{m^2-1}{4},\ B=\frac{m^2+1}{2},\ C=\frac{m^2-1}{4}$. (38)

$$u_{14}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}\left(\frac{dn(\xi)}{1\pm msn(\xi)}\right)^2+b_2\left(\frac{1\pm msn(\xi)}{dn(\xi)}\right)^2.$$

$$v_{14}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C\left(\frac{dn(\xi)}{1\pm msn(\xi)}\right)^2+2A\left(\frac{1\pm msn(\xi)}{dn(\xi)}\right)^2.$$

x) If $A=\frac{1-m^2}{4},\ B=\frac{m^2+1}{2},\ C=\frac{1-m^2}{4}$. (39)

$$u_{15}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}\left(\frac{cn(\xi)}{1\pm sn(\xi)}\right)^2+b_2\left(\frac{1\pm sn(\xi)}{cn(\xi)}\right)^2.$$

$$v_{15}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C\left(\frac{cn(\xi)}{1\pm sn(\xi)}\right)^2+2A\left(\frac{1\pm sn(\xi)}{cn(\xi)}\right)^2.$$

xi) If $A=-\frac{(1-m^2)^2}{4},\ B=\frac{m^2+1}{2},\ C=-\frac{1}{4}$. (40)

$$u_{16}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}(mcn(\xi)\pm dn(\xi))^2+b_2\left(\frac{1}{mcn(\xi)\pm dn(\xi)}\right)^2.$$

$$v_{16}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C(mcn(\xi)\pm dn(\xi))^2+2A\left(\frac{1}{mcn(\xi)\pm dn(\xi)}\right)^2.$$

xii) If $A=\frac{1}{4},\ B=\frac{m^2+1}{2},\ C=\frac{(1-m^2)^2}{4}$. (41)

$$u_{17}=\frac{Bb_2+b_2\sqrt{B^2+12AC}}{3A}+\frac{b_2C}{A}\left(\frac{sn(\xi)}{dn(\xi)\pm cn(\xi)}\right)^2+b_2\left(\frac{dn(\xi)\pm cn(\xi)}{sn(\xi)}\right)^2.$$

$$v_{17}=\frac{2}{3}\left(B+\sqrt{B^2+12AC}\right)+2C\left(\frac{sn(\xi)}{dn(\xi)\pm cn(\xi)}\right)^2+2A\left(\frac{dn(\xi)\pm cn(\xi)}{sn(\xi)}\right)^2.$$

xiii) If $A=\frac{1}{4},\ B=\frac{m^2-2}{2},\ C=\frac{m^4}{4}$. (42)

$$u_{18} = \frac{Bb_2 + b_2\sqrt{B^2 + 12AC}}{3A} + \frac{b_2 C}{A}\left(\frac{cn(\xi)}{\sqrt{1-m^2} \pm dn(\xi)}\right)^2 + b_2\left(\frac{\sqrt{1-m^2} \pm dn(\xi)}{cn(\xi)}\right)^2.$$

$$v_{18} = \frac{2}{3}\left(B + \sqrt{B^2 + 12AC}\right) + 2C\left(\frac{cn(\xi)}{\sqrt{1-m^2} \pm dn(\xi)}\right)^2 + 2A\left(\frac{\sqrt{1-m^2} \pm dn(\xi)}{cn(\xi)}\right)^2.$$

where, $\xi = x + \frac{b_2}{2A} y + 4\sqrt{B^2 + 12AC}t.$

Here $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ are Jacobi elliptic functions and m denotes the modulus of the Jacobi elliptic functions.
If $m \to 1,$ then $sn\xi \to \tanh\xi,\ cn\xi \to \sec h\xi,\ dn\xi \to \sec h\xi.$ We can obtain the following some multiple soliton solutions of Eq. (24).

$$u_{19} = \frac{2b_2}{3} + b_2 \tanh^2(\xi) + b_2\left(\frac{1}{\tanh^2(\xi)}\right), \quad v_{19} = \frac{4}{3} + 2\tanh^2(\xi) + 2\left(\frac{1}{\tanh^2(\xi)}\right).$$

where, $\xi = x + \frac{b_2}{2} y + 16t.$

$$u_{20} = \frac{2}{3}b_2 + b_2\left(\frac{\tanh(\xi)}{1 \pm \sec h(\xi)}\right)^2 + b_2\left(\frac{1 \pm \sec h(\xi)}{\tanh(\xi)}\right)^2, \quad v_{20} = \frac{1}{3} + \frac{1}{2}\left(\frac{\tanh(\xi)}{1 \pm \sec h(\xi)}\right)^2 + \frac{1}{2}\left(\frac{1 \pm \sec h(\xi)}{\tanh(\xi)}\right)^2.$$

where, $\xi = x + 2b_2 y + 4t.$

If $m \to 0,$ then $sn\xi \to \sin\xi,\ cn\xi \to \cos\xi,\ dn\xi \to 1.$ We can obtain triangular periodic solutions of Eq. (24). The triangular periodic solutions of Eq. (24) are omitted.

6. CONCLUSION AND SUGGESTIONS (SONUÇ VE ÖNERİLER)

In this paper, we present the Generalized Jacobi elliptic function method by using ansatz (3) and, with aid of Mathematica, implement it in a computer algebraic system. An implementation of the method is given by applying it to KdV equation and (2+1)-dimensional coupled KdV system, we also obtain some new double-periodic solutions and multiple soliton solutions at same time. We can obtain not only the double periodic solutions, but also triangular periodic solutions as well as the multiple soliton solutions of the KdV equation and (2+1)-dimensional coupled KdV system. The method can be used to many other nonlinear equations or coupled ones. In addition, this method is also computerizable, which allows us to perform complicated and tedious algebraic calculation on a computer.

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