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THE IMPORTANCE OF PROBLEM SOLVING IN MATHEMATICS CURRICULUM

ABSTRACT

The basic purpose of this research is to present how to solve a mathematics problem or real problem. In this article, we point out the goals of problem solving in Mathematics Curriculum. This study contains problem solving literature in the world and new studies for problem solving education "Center of Mathematics Education", especially, "Art of Problem Solving". In this study we try to explain our opinions on the strength of our observation in the classrooms.

Keywords: Mathematics, Mathematics Education,
Mathematics Curriculum, Importance of Problem Solving,
Ways of Problem Solving

PROBLEM ÇÖZMENİN MATEMATİK MÜFREDATINDAKİ ÖNEMİ

ÖZET

Bu araştırmanın en önemli amacı bir matematik probleminin veya gerçek bir problemin nasıl çözüleceğini ortaya koymaktır. Bu makalede Matematik müfredatında bulunan problem çözmenin amaçlarını belirtiyoruz. Bu çalışma dünyadaki problem çözme literatürünü ve "Matematik Eğitim Merkezi" olarak anılan problem çözme eğitiminini özellikle "Problem Çözme Sanatı" olarak içeriyor. Bu çalışmada düşüncelerimizi daha çok sınıflardaki gözlemlerimize dayanarak açıklamaya çalışıyoruz.

Keywords: Matematik, Matematik Eğitimi, Matematik Müfredatı,
Problem Çözmenin Önemi, Problem Çözme Yolları



1. INTRODUCTION (GİRİŞ)

Problem Solving is about solving problems. And we'll restrict ourselves to thinking about mathematical problems here even though problem solving in school has a wider goal. When you think about it, the whole aim of education is to equip children to solve problems, but not only in math. Aim of the new Mathematics Curriculum in Turkey therefore, problem solving contributes to the generic skill of problem solving in daily life.

The ultimate goal of any problem-solving program is to improve students' performance at solving problems correctly. The specific goals of problem-solving in Mathematics are to:

- Improve students' willingness to try problems and improve their perseverance when solving problems.
- Improve students' self-concepts with respect to the abilities to solve problems.
- Make students aware of the problem-solving strategies.
- Make students aware of the value of approaching problems in a systematic manner.
- Make students aware that many problems can be solved in more than one way.
- Improve students' abilities to select appropriate solution strategies.
- Improve students' abilities to implement solution strategies accurately.
- Improve students' abilities to get more correct answers to problems [8].

2. RESEARCH SIGNIFICANCE (ÇALIŞMANIN ÖNEMİ)

In this study, we searched problem solving literature, not only in Turkey's Mathematics Education, in the other countries' Mathematics Education, too. In this wise, we can see the importance of problem solving in Mathematics Curriculum in the world. We wanted to see how problem solving is to be affected from new developments in Mathematics Education. And we wanted to see effects of new teaching approaches to students' learnings in problem solving education. At the same time, we explain how the students must use ways of problem solving.

3. THE STRATEGIES OF PROBLEM SOLVING (PROBLEM ÇÖZME STRATEJİLERİ)

Problem Solving also contributes to mathematics itself. It is part of one whole area of the subject that has largely passed unnoticed in schools around the world. Mathematics consists of skills and processes. The skills are things that we are all familiar with. These include the basic arithmetical processes and the algorithms that go with them. They include algebra in all its levels as well as sophisticated areas such as the calculus. This is the side of the subject that is largely represented in the Strands of Number, Algebra, Statistics, Geometry and Measurement [8].

On the other hand, the processes of mathematics are the ways of using the skills creatively in new situations. Problem solving is a mathematical process. As such it is to be found in the Strand of Mathematical Processes along with Logic and Reasoning, and Communication. This is the side of mathematics that enables us to use the skills in a wide differently of situations.



A problem is a task for which:

- The person confronting it wants or needs to find a solution.
- The person has no readily available procedure for finding the solution.
- The person must make an attempt to find a solution [8].

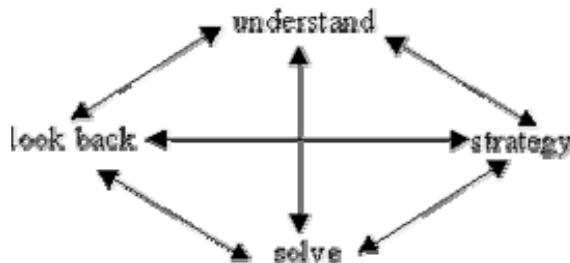
Problem Solving is worth pointing out that we find it useful to distinguish between the three words "method", "answer" and "solution". By "method" we mean the means used to get an answer. This will generally involve one or more problem solving strategies. On the other hand, we use "answer" to mean a number, quantity or some other entity that the problem is asking for. Finally, a "solution" is the whole process of solving a problem, including the method of obtaining an answer and the answer itself. We must distinguish every detail in problem solving. "**method + answer = solution**".

Problem solving has not rules, but has a systematic [1].

In 1973 Pólya enunciated four basic steps to solve problems;

- Understand and explore the problem,
- Find a strategy,
- Use the strategy to solve the problem,
- Look back and reflect on the solution.

This four step is the most popular way of problem solving [1]. Although we have listed the Four Stages of Problem Solving in order, for difficult problems it may not be possible to simply move through them consecutively to produce an answer. It is frequently the case that children move backwards and forwards between and across the steps. In fact the diagram below is much more like what happens in practice.



Graph 1. Rotation of problem solving
 (Grafik 1. Problem çözme döngüsü)

There is no chance of being able to solve a problem unless you can understand it first. This process requires not only knowing what you have to find but also the key pieces of information that somehow need to be put together to obtain the answer.

People will often not be able to absorb all the important information of a problem in one go. It will almost always be necessary to read a problem several times, both at the start and during working on it. During the solution process, people may find that they have to look back at the original question from time to time to make sure that they are on the right track. We can ask these questions in this stage:

- What is the unknown?
- What are the data?
- What is the condition? [11].

Pólya's second stage of finding a strategy tends to suggest that it is a fairly simple matter to think of an appropriate strategy. However, there are certainly problems where children may find it necessary to play around with the information before they are able to think of a strategy that might produce a solution. This exploratory



phase will also help them to understand the problem better and may make them aware of some piece of information that they had neglected after the first reading. We must do these in this stage: Do I know a related problem?, try to think of a familiar problem having the same or a similar unknown.

Some problems are too hard so it is necessary to give up. You may give up so that you can take a rest, in which case it is a 'for now' giving up. I think this is a very good problem solving strategy and must use in Mathematics Education. Often when you give up for a while your subconscious takes over and comes up with a good idea that you can use. On the other hand, some problems are so hard that you eventually have to give up 'for ever'. There have been many difficult problems throughout history that mathematicians have had to give up on.

The most important thing in solving problem is students' beliefs of nature of mathematics [4]. Teachers must teach the nature of mathematics and give confidence to students to solve the questions.

That then is a rough overview of what Problem Solving is all about. For simple problems the four stage Pólya method and the scientific method can be followed through without any difficulty [9]. But when the problem is hard we can give up for a while and then come with new ideas to solve problems.

Bird says that when you want to solve a problem, you must ask that question: "What would happen if...." [2]. This questions shows us right answer what we can do. And when we begin to ask this question in all problems, we are on true way.

4. THE IMPORTANCE OF PROBLEM SOLVING (PROBLEM ÇÖZMENİN ÖNEMLİ)

Having explored the problem and decided on a plan of attack, the third problem-solving step, solve the problem, can be taken. Hopefully now the problem will be solved and an answer obtained. During this phase it is important for the children to keep a track of what they are doing. This is useful to show others what they have done and it is also helpful in finding errors should the right answer not be found. We must do these in this stage: see clearly that the step is correct, prove that the step is correct. At this point many people, especially mathematically able ones, will stop. But it is worth getting them into the habit of looking back over what they have done. There are several good reasons for this. The most important habit of looking back is people are often able to see another way of solving the problem, for all that we can accustomed to use new way in our real problems.

This new solution may be a nicer solution than the original and may give more insight into what is really going on. Eventually, successfully students especially, may be able to generalise or extend the problem. We must do these in this stage: check the result, check the argument, derive the result differently, and see it at a glance.

Generalising a problem means creating a problem that has the original problem as a special case. So when we generalize a problem, unconsciously, we take the formula that consists some methods of real problems solving. It gives us a confidence regarding to success in math or daily problems. For example, if we have some sort of Formula for any height then we can substitute into that formula to get the answer for height of some shapes.

Extending a problem is a related idea. This brings us to an aspect of problem solving that we haven't mentioned so far. This is proof. Your students may often be able to guess what the answer to a problem is but their solution is not complete until they can justify their answer. In some problems it is hard to find a justification. Indeed you may believe that it is not something that any of the class



can do. So you may be happy that the children can guess the answer. However, bear in mind that this justification is what sets mathematics apart from every other discipline. Consequently the justification step is an important one that shouldn't be missed too often.

Here the problem is given and initially the idea is to experiment with it or explore it in order to get some feeling as to how to proceed. After a while it is hoped that the solver is able to make a conjecture or guess what the answer might be. If the conjecture is true it might be possible to prove or justify it. In that case the looking back process sets in and an effort is made to generalise or extend the problem. In this case you have essentially chosen a new problem and so the whole process starts over again.

Some mathematicians say problem solving is at the heart of Mathematics. In that case problem solving must be the center of the Mathematics Curriculum. In 1999, research of international research group TIMMS shows us Turkey is the 31th country in 39 countries at in Math success. I think the reason of this is, curriculum did not focus on problem solving. But new Mathematics Curriculum is better than the old done for problem solving. It gives a chance to students to solve different problems by one. It is all about confidence to pupil. We can prepare pupils to real world to solve more and more real (non-routine) problems.

It has been suggested that problem-solving techniques can be made available most effectively through making problem solving the focus of the mathematics curriculum. Although mathematical problems have traditionally been a part of the mathematics curriculum, it has been only comparatively recently that problem solving has come to be regarded as an important medium for teaching and learning mathematics [14]. In the past problem solving had a place in the mathematics classroom, but it was usually used in a token way as a starting point to obtain a single correct answer, usually by following a single 'correct' procedure. More recently, however, professional organisations such as the National Council of Teachers of Mathematics have recommended that the mathematics curriculum should be organized around problem solving, focusing on:

- Developing skills and the ability to apply these skills to unfamiliar situations,
- Gathering, organising, interpreting and communicating information,
- Formulating key questions, analyzing and conceptualizing problems, defining problems and goals, discovering patterns and similarities, seeking out appropriate data, experimenting, transferring skills and strategies to new situations,
- Developing curiosity, confidence and open-mindedness [10].

According to Resnick a problem-solving approach contributes to the practical use of mathematics by helping people to develop the facility to be adaptable when, for instance, technology breaks down. It can thus also help people to transfer into new work environments at this time when most are likely to be faced with several career changes during a working lifetime [8]. Resnick expressed the belief that 'school should focus its efforts on preparing people to be good adaptive learners, so that they can perform effectively when situations are unpredictable and task demands change'.

Many of Mathematicians advance that 'Mathematics is an art', so I can say if Mathematics is an art, 'heart of Mathematics' problem solving is an art, too.

Problem solving is an important component of mathematics education because it is the single vehicle which seems to be able to



achieve at school level all three of the values of mathematics listed at the outset of this article: functional, logical and aesthetic. Let us consider how problem solving is a useful medium for each of these.

It has already been pointed out that mathematics is an essential discipline because of its practical role to the individual and society. Through a problem-solving approach, this aspect of mathematics can be developed. Presenting a problem and developing the skills needed to solve that problem is more motivational than teaching the skills without a context. Such motivation gives problem solving special value as a vehicle for learning new concepts and skills or the reinforcement of skills already acquired [14]. Approaching mathematics through problem solving can create a context which simulates real life and therefore justifies the mathematics rather than treating it as an end in itself. The National Council of Teachers of Mathematics recommended that problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life [10]. Furthermore it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives. More recently the Council endorsed this recommendation with the statement that problem solving should underly all aspects of mathematics teaching in order to give students experience of the power of mathematics in the world around them. They see problem solving as a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others.

Cockcroft also advocated problem solving as a means of developing mathematical thinking as a tool for daily living, saying that problem-solving ability lies 'at the heart of mathematics' because it is the means by which mathematics can be applied to a variety of unfamiliar situations [5].

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill which can enhance logical reasoning. Individuals can no longer function optimally in society by just knowing the rules to follow to obtain a correct answer. They also need to be able to decide through a process of logical deduction what algorithm, if any, a situation requires, and sometimes need to be able to develop their own rules in a situation where an algorithm cannot be directly applied. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking [10] rather than just as the means to an end of finding the correct answer.

Many writers have emphasised the importance of problem solving as a means of developing the logical thinking aspect of mathematics. 'If education fails to contribute to the development of the intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems ... '[12].

Modern definitions of intelligence talk about practical intelligence which enables 'the individual to resolve genuine problems or difficulties that he or she encounters' and also encourages the individual to find or create problems 'thereby laying the groundwork for the acquisition of new knowledge' [6]. As was pointed out earlier, standard mathematics, with the emphasis on the acquisition of knowledge, does not necessarily cater for these needs.

Resnick described the discrepancies which exist between the algorithmic approaches taught in schools and the 'invented' strategies which most people use in the workforce in order to solve practical problems which do not always fit neatly into a taught algorithm [13]. Most people have developed 'rules of thumb' for calculating, for



example, quantities, discounts or the amount of change they should give, and these rarely involve standard algorithms. Training in problem-solving techniques equips people more readily with the ability to adapt to such situations.

A further reason why a problem-solving approach is valuable is as an aesthetic form. Problem solving allows the student to experience a range of emotions associated with various stages in the solution process. Mathematicians who successfully solve problems say that the experience of having done so contributes to an appreciation for the 'power and beauty of mathematics' [14] the "joy of banging your head against a mathematical wall, and then discovering that there might be ways of either going around or over that wall". Whole school year observe changes in the pupils' ability to be able to transfer knowledge to unfamiliar situations.

"Real Problem Solving" enables pupils to practise their mathematics skills or learn new ones, by using mathematics as a tool to address a problem. Have your pupils identify and explore a solution to a problem. Identify the mathematical skills which are used, and the values which are encouraged. Discuss with your pupils how their project has been useful in helping others.

For many reasons, the state of society has reached a stage where it is more critical than ever to educate people in the traditional values of their culture. In recent years there has been considerable discussion about whether it is the responsibility of schools to impart values education. There is growing pressure for all teachers to become teachers of values.

Problem solving has the major tasks; preparing students for the challenges of life, encouraging the development of general knowledge and common sense and engaging students in service to society.

Sometimes pupils can be asked to make up their own problems, which can help to enhance their understanding. This can encourage them to be flexible, and to realise that there can be more than one way of looking at a problem. Further, the teacher can set a theme for the problems that the pupils make up, such as giving help to others or concern for the environment, which can help them to focus on the underlying values as well as the mathematics.

Bohan, Irby and Vogel suggest a seven-step model for doing research in the classroom, to enable students to become "producers of knowledge rather than merely consumers" [3].

Their findings may contribute to the knowledge base of the class, the school, the community, or society as a whole. "Their findings may affect their school or their world in a very positive way" [3].

Bird suggests that an investigation approach is suitable for many topics in the curriculum and encourages communication, confidence, motivation and understanding as well as mathematical thinking [2].

5. CONCLUSION AND SUGGESTIONS (SONUÇ VE ÖNERİLER)

There are many reasons why a problem-solving approach can contribute significantly to the outcomes of a mathematics education. Not only is it a vehicle for developing logical thinking, it can provide students with a context for learning mathematical knowledge, it can enhance transfer of skills to unfamiliar situations and it is an aesthetic form in itself. A problem-solving approach can provide a vehicle for students to construct their own ideas about mathematics and to take responsibility for their own learning.

While these are all important mathematics skills, they are also important life skills and help to expose pupils to a values education



that is essential to their holistic development. The National Council of Teachers of Mathematics (NCTM) recommended that problem solving be the focus of mathematics teaching because, they say, it encompasses skills and functions which are an important part of everyday life [10]. There is not a specific solving method in all problems as a matter of fact problem solving is the most mixed subject in Math. Problem solving just requires practice! The more you practice, the better you get. And problem solving must be in the center of the Mathematics Curriculum. At last I say problem solving is the best 'art' in the world.

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